Exam solution for PCHEM 3060

17th October 2005

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(1.) For what reason(s)are the energies of the quantum mechanical systems quantized?

Here i wanted to hear: ***Boundary conditions!!!*** *But still if you wrote sth. about quantized Energy levels and it was correct, i still gave you 2Pts for that!*

(2.) Describe Heisenberg's Uncertainty principle.

Well, you can't measure the position and momentum of a particle simultanously with absolute precision.

(3.) Which motion does a harmonic oscillator describe in molecules.

Vibrational motions

(4.) What is degeneracy? Give an example.

Degeneracy is if you can describe different states possess the same energy value..for instance the p-orbitals.

(5.) List three facts about quantum mechanics that are in contradiction with the corresponding classical mechanics.

Here, you could have mentioned Black body radiation, Heat capacity, Photoelectric effect as well as Heisenbergs uncertainty principle....

(5.) Since $\hat{p_x}$ and $\hat{p_y}$ coummute, every eigenfunction of $\hat{p_x}$ is also an eigenfunction of $\hat{p_y}$.

True [False]

(6.) Eigenfunctions of the same operator that have different eigenvalues are always orthogonal.

[True] False

(7.) $[\hat{L^2}, \hat{L_x}]$,$[\hat{L^2}, \hat{L_y}]$,$[\hat{L^2}, \hat{L_z}]$ all commute and therefore$\hat{L^2}, \hat{L_x}.\hat{L_y} and \hat{L_z}$ can be mesured simultaneously with absolute certainty.

[True] [False]

(8.) The threshold wavelength for potassium metal is 564 nm. What is its work function? What is the kinetic energy of electrons ejected if radiation of wavelength 410 nm is used?

$$\nu_0 = \frac{c}{\lambda_0} = \frac{2.998 * 10^8 m * s^- 1}{564 * 10^- 9 * m} = 5.32 * 10^{14} Hz$$

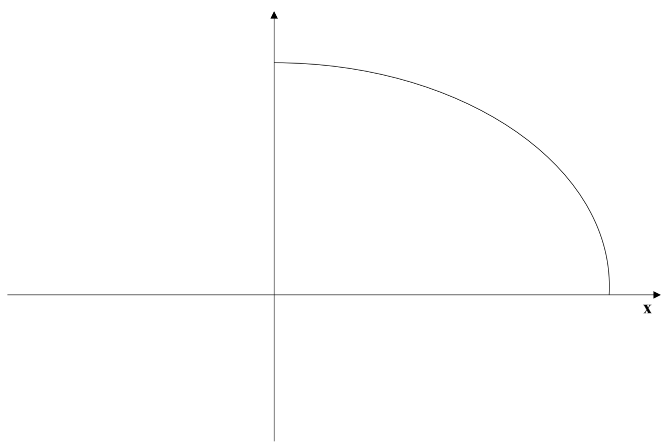
$$\Phi = h * \nu_0 = (6.626 * 10^{-34} Js) * (5.32 * 10^{14} Hz) = \underline{3.52 * 10^{-19} J}$$

$$KE = h*\nu - \Phi = \frac{h * c}{\lambda} - \Phi = \frac{6.626 * 10^{-34} Js 2.998 * 10^8}{410 * 10^{-9}} - 3.52*10^{-19} J = \underline{1.32 * 10^{-19} J}$$

1. A particle of mass μ is constrained to move **on a portion** of a ring of radius r_0, according to the potential:

$$V(Phi) = 0 \qquad for 0 \leq \Phi \leq \frac{\pi}{2}$$
$$V(Phi) = \infty \qquad for\ \phi\ outside\ of\ this\ range.$$

You will note that this is one-quarter of the full ring, corresponding to the portion of the ring in just one quadrant:



The Schrödinger equation for this system is:

$$-\frac{\hbar^2}{2\mu r_0^2} \frac{d^2\Psi}{d\Phi^2} = E\Psi$$

The general solution for this equation(before applying the boundary conditions) is

$$\Psi(\Phi) = c_1 sin\left(\sqrt{\frac{2\mu E r^2}{\hbar^2}}\Phi\right) + c_2 cos\left(\sqrt{\frac{2\mu E r^2}{\hbar^2}}\Phi\right)$$

(a) What are the boundary conditions that must be placed on this system?

$$\Psi(0) = 0 \qquad and \qquad \Psi(\frac{\pi}{2}) = 0$$

(b) Apply the boundary conditions to find the energy of the system and the expression for the wavefunction.

$$\Psi(\Phi) = c_1 sin\left(\sqrt{\frac{2\mu E r^2}{\hbar^2}}\Phi\right) + c_2 cos\left(\sqrt{\frac{2\mu E r^2}{\hbar^2}}\Phi\right)$$

$$\Psi(0) = c_1 * \underbrace{sin(0)}_{=0} + c_2 * \underbrace{cos(0)}_{=1}$$

So it turns out that in order to get the wavefunction to zero, c_2 must be zero!

$$\Psi(\frac{\pi}{2}) = c_1 * sin\left(\sqrt{\frac{2 * \mu * E * r^2}{\hbar^2}} * \frac{\pi}{2}\right) = 0$$

$$\sqrt{\frac{2 * \mu * E * r^2}{\hbar^2}} * \frac{\pi}{2} \doteq n * \pi$$

$$\frac{2 * \mu * E * r^2}{\hbar^2} = 4 * n^2$$

$$\Rightarrow \boxed{E = \frac{2n^2 * \hbar^2}{\mu * r^2}}$$

and

$$\boxed{\Psi(\phi) = c_1(sin(2n\phi)) with} \; n = 1, 2, 3...$$

(c) Set up to solve for the normalization constant for the ground state wave function.

$$\boxed{\left|c_1^2\right| \int_0^{\frac{\pi}{2}} (sin^2 2n\phi) d\phi = 1}$$

(d) Set up for calculating the average ϕ for the ground state (You don't need to perform the integral).

$$\boxed{\phi = \int_0^{\frac{\pi}{2}} ((sin 2n\phi)^* * \phi * (sin 2n\phi)) d\phi}$$